Polar Area

19. The area of the region inside the polar curve $r = 4\sin\theta$ and outside the polar curve r = 2 is given by

$(A)\ \frac{1}{2}\int_0^{\pi}(4\sin\theta - 2)^2\,d\theta$ $\quad (B)\ \frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}(4\sin\theta - 2)^2\,d\theta$ $\quad (C)\ \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}}(4\sin\theta - 2)^2\,d\theta$

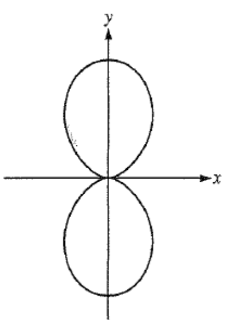
$(D)\ \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}}(16\sin^2\theta - 4)\,d\theta$ $\quad (E)\ \frac{1}{2}\int_0^{\pi}(16\sin^2\theta - 4)\,d\theta$

21. Which of the following is equal to the area of the region inside the polar curve $r = 2\cos\theta$ and outside the polar curve $r = \cos\theta$?

$(A)\ 3\int_0^{\frac{\pi}{2}}\cos^2\theta\,d\theta$ $\quad (B)\ 3\int_0^{\pi}\cos^2\theta\,d\theta$ $\quad (C)\ \frac{3}{2}\int_0^{\frac{\pi}{2}}\cos^2\theta\,d\theta$

$(D)\ 3\int_0^{\frac{\pi}{2}}\cos\theta\,d\theta$ $\quad (E)\ 3\int_0^{\pi}\cos\theta\,d\theta$

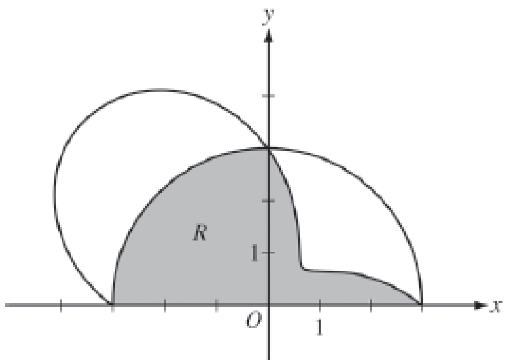
26. Which of the following expressions gives the total area enclosed by the polar curve $r = \sin^2\theta$ shown in the figure



?

A) $\frac{1}{2}\int_0^{\pi}\sin^2\theta d\theta$ $\quad$ B) $\int_0^{\pi}\sin^2\theta d\theta$ $\quad$ C) $\frac{1}{2}\int_0^{\pi}\sin^4\theta d\theta$

D) $\int_0^{\pi}\sin^4\theta d\theta$ $\quad$ E) $2\int_0^{\pi}\sin^4\theta d\theta$



2. The graphs of the polar curves r = 3 and $r = 3 - 2\sin(2\theta)$ are shown in the figure above for $0 \le \theta \le \pi$.

a) Let R be the shaded region that is inside the graph of r = 3 and inside the graph of $r = 3 - 2\sin(2\theta)$. Find the area of R.

b) For the curve $r = 3 - 2\sin(2\theta)$, find the value of $\frac{dx}{d\theta}$ at $\theta = \frac{\pi}{6}$.

c) The distance between the two curve changes for $0 \le \theta \le \frac{\pi}{2}$. Find the rate at which the distance between the two curves is changing with respect to θ when $\theta = \frac{\pi}{3}$.

d) A particle is moving along the curve $r = 3 - 2\sin(2\theta)$ so that $\frac{d\theta}{dt} = 3$ for all times $t \ge 0$. Find the value of $\frac{dr}{dt}$ at $\theta = \frac{\pi}{6}$.

2011 Form B

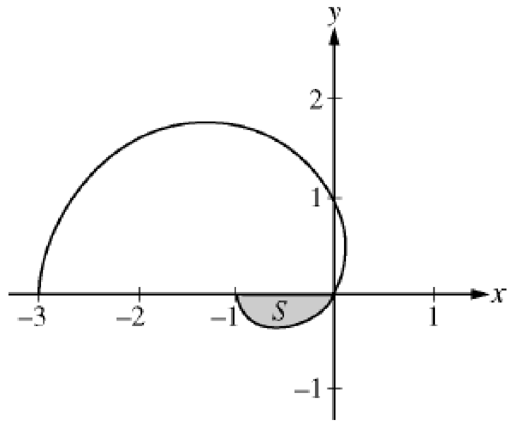
2. The polar curve r is given $r = 3\theta + \sin\theta$, where $0 \leq \theta \leq 2\pi$.

a) Find the area in the second quadrant enclosed by the coordinate axes and the graph of r.

b) For $\frac{\pi}{2} \leq \theta \leq \pi$, there is one point P on the polar curve with x-coordinate -3. Find the angle θ that corresponds to point P. Find the y-coordinate of point P. Show the work that leads to your answers.

c) A particle is traveling along the polar curve r so that its position at time t is (x(t), y(t)) and such that $\frac{d\theta}{dt} = 2$. Find $\frac{dy}{dt}$ at the instant $\theta = \frac{2\pi}{3}$, and interpret the meaning of your answer in the context of the problem.

2009 Form B BC 4



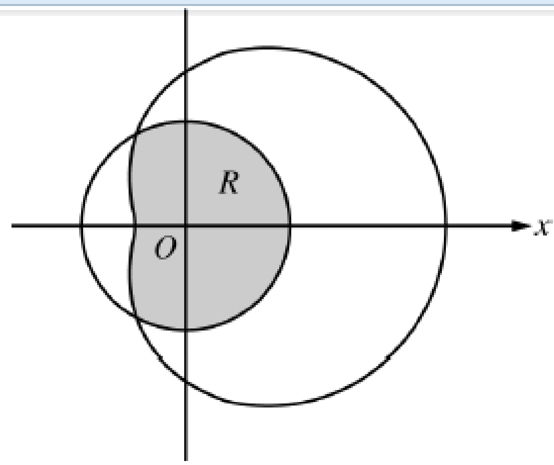
4. The graph of the polar curve $r = 1 - 2\cos\theta$ for $0 \le \theta \le \pi$ is shown above. Let S be the shaded region in the third quadrant bounded by the curve and the x-axis.

a) Write an integral expression for the area of S.

b) Write expressions for $\frac{dx}{d\theta}$ and $\frac{dy}{d\theta}$ in terms of θ.

c) Write an equation in terms of x and y for the line tangent to the graph of the polar curve at the point $\theta = \frac{\pi}{2}$. Show the computations that lead to your answer.

2007 BC 3

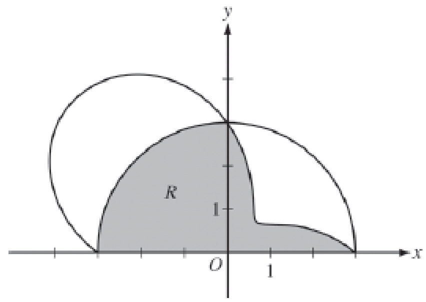


3. The graphs of the polar curves r = 2 and $r = 3 + 2\cos\theta$ are shown in the figure above. The curves intersect when $\theta = \frac{2\pi}{3}$ and $\theta = \frac{4\pi}{3}$.

a) Let R be the region that is inside the graph of r = 2 and also inside the graph of $r = 3 + 2\cos\theta$, as shaded in the figure above. Find the area of R.

b) A particle moving with nonzero velocity along the polar curve given by $r = 3 + 2\cos\theta$ has position (x(t), y(t)) at time t, with $\theta = 0$ when t = 0. This particle moves along the curve so that $\frac{dr}{dt} = \frac{dr}{d\theta}$. Find the value of $\frac{dr}{dt}$ at $\theta = \frac{\pi}{3}$ and interpret your answer in terms of the motion of the particle.

c) For the particle described in part b, $\frac{dy}{dt} = \frac{dy}{d\theta}$. Find the value of $\frac{dy}{dt}$ at $\theta = \frac{\pi}{3}$ and interpret your answer in terms of the motion of the particle.

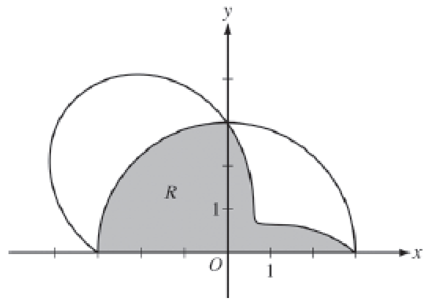


2. The graphs of the polar curves r = 3 and $r = 3 - 2\sin(2\theta)$ are shown in the figure above for $0 \le \theta \le \pi$.

a) Find the slope at $\theta = \frac{\pi}{3}$ for the curve $r = 3 - 2\sin(2\theta)$

b) A particle moving with nonzero velocity along the polar curve given by $r = 3 - 2\sin(2\theta)$ has position (x(t), y(t)) at time t, with $\theta = 0$ when t = 0. This particle moves along the curve so that $\frac{dr}{dt} = \frac{dr}{d\theta}$. Find the value of $\frac{dr}{dt}$ at $\theta = \frac{\pi}{3}$ and interpret your answer in terms of the motion of the particle.

c) For the curve $r = 3 - 2\sin(2\theta)$, find the value of $\frac{dx}{d\theta}$ at $\theta = \frac{\pi}{6}$.



d) The distance between the two curve changes for $0 \le \theta \le \frac{\pi}{2}$. Find the rate at which the distance between the two curves is changing with respect to θ when $\theta = \frac{\pi}{3}$.

e) For $0 \le \theta \le \pi$, there is one point P on the polar curve with x-coordinate -2. Find the angle θ that corresponds to point P. Find the y-coordinate of point P. Show the work that leads to your answers.

f) A particle is moving along the curve $r = 3 - 2\sin(2\theta)$ so that $\frac{d\theta}{dt} = 3$ for all times $t \ge 0$. Find the value of $\frac{dr}{dt}$ at $\theta = \frac{\pi}{6}$.